Stress Field and Fracturing of Solid Propellant during Rocket Firing

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The occurrence of crazing, cracking, or fracturing in viscoelastic materials has been attributed to its strong association with the presence of tensile stress.[1,2] In the problem of studying fracturing of solid propellant during rocket firing, the determination of the stress distribution in the solid propellant must be made as a prerequisite. However, because of the development of the anisotropic properties of the solid propellant as a result of microscopic nonhomogeneity in its constituents, the stress analysis must be based upon anisotropic viscoelastic considerations.[3] Using general indicial notations and summation convention over repeated diagonal indices, the anisotropic constitutive equations in linear infinitesimal viscoelasticity may be deduced from invariant considerations.[4]

Under certain restrictions the stress tensor $\sigma^{ij}(x^k,t)$ at time t can be related to the history of the infinitesimal strain tensor $\epsilon_{ij}(x^k,t-\tau)$ where $0 \leq \tau \leq \infty$, i.e.,

$$\sigma^{ij}(x^k,t) = \int_{-\infty}^{t} c^{ijkl}(t-\tau)\frac{\partial}{\partial\tau}\epsilon_{kl}(x^k,\tau)d\tau \qquad (1)$$

where x^k are space variables and $C^{ijkk}(t)$ are anisotropic relaxation moduli of the viscoelastic material.[5] Employing the integral transform technique in the form of Laplace transforms, the governing differential equation for an anisotropic viscoelastic long cylinder under symmetric uniform unit step internal pressure $p_0H(t)$ and free outer boundary can be solved. In cylindrical polar coordinates (x,θ,z) where x is nondimensional radial coordinate, the radial and tangential stress fields are found as follows[6]

$$\sigma^{xx}(x^k,t) = -\frac{p_0}{x}\frac{\sinh[\bar{K}(0)\ln(x_2/x)]}{\sinh[\bar{K}(0)\ln x_2]}H(t) + \frac{2\pi p_0}{x(\ln x_2)^2}\sum_{\eta=1}^{L}\sum_{m=1}^{\infty}\frac{(-1)^m m\sin\left(m\pi\dfrac{\ln(x_2/x)}{\ln x_2}\right)e^{-\lambda_\eta^{(m)}t}}{\left[f'(s)+\dfrac{\phi'(s)}{g(s)}-\dfrac{\phi(s)g'(s)}{[g(s)]^2}\right]_{s=-\lambda_\eta^{(m)}}} \qquad (2)$$

and

$$\sigma^{\theta\theta}(x^k,t) = \frac{p_0\bar{K}(0)}{x}\frac{\cosh[\bar{K}(0)\ln(x_2/x)]}{\sinh[\bar{K}(0)\ln x_2]}H(t) - \frac{2\pi^2 p_0}{x(\ln x_2)^3}\sum_{\eta=1}^{L}\sum_{m=1}^{\infty}\frac{(-1)^m m^2\cos\{m\pi[\ln(x_2/x)/\ln x_2]\}e^{-\lambda_\eta^{(m)}t}}{\left[f'(s)+\dfrac{\phi'(s)}{g(s)}-\dfrac{\phi(s)g'(s)}{[g(s)]^2}\right]_{s=-\lambda_\eta^{(m)}}} \qquad (3)$$

where

$$\bar{K}(s) \equiv \frac{\bar{C}^{\theta\theta\theta\theta}(s)}{\bar{C}^{xxxx}(s)} = \left[f(s)+\frac{\phi(s)}{g(s)}\right]^{1/2} \qquad (4)$$

and $\bar{K}(0) = \bar{K}(s)$ evaluated at $s = 0$. x_2 is the external boundary of the cylinder and, $-\lambda_\eta^{(m)}$ are the roots of the polynomials of degree L in s, i.e.,

$$f(s)g(s) + \phi(s) + [m^2\pi^2/(\ln x_2)^2]g(s) = 0 \qquad (5)$$

where $f(s)$, $g(s)$, and $\phi(s)$ are polynomials in s. The primed functions denote derivatives of the functions with respect to Laplace parameter s.

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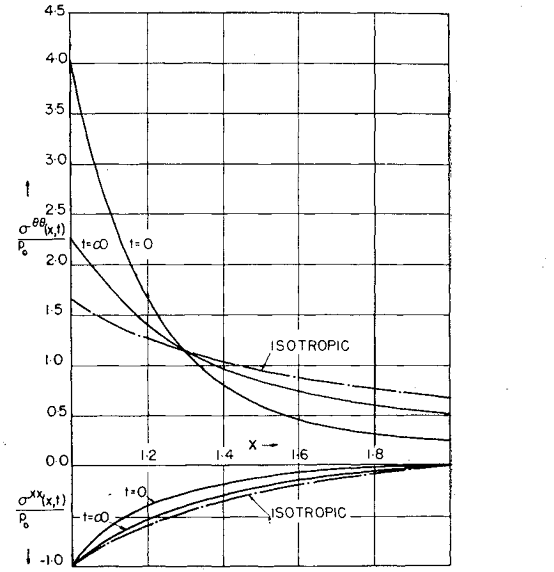

Fig. 1 Tangential and radial stress distribution in the viscoelastic cylinder.

Both short-time and long-time behaviors of the radial and tangential stress fields were evaluated for a mathematically simple viscoelastic material. Numerical results are presented here for the case of a typical anisotropic rubber medium[7] with modulus of elasticity ratio $E^{\theta\theta\theta\theta}/E^{xxxx} = 4$ and an assumed value of 4 for $\tau^{\theta\theta\theta\theta}/\tau^{xxxx}$ in the relation

$$\bar{C}^{kkkk}(s) = E^{kkkk}[(1/s) + \tau^{kkkk}]$$

where k is x or θ. This is equivalent to considering the simplest viscoelastic behavior in creep. The results of radial and tangential stress fields for a cylinder with outside radius twice as long as the inside radius are shown in Fig. 1. A significant variation of the tangential stress field is seen to be in the cylinder close to its inner boundary. It appears that the tangential stress field is tensile, and its magnitude can occur at a level many times greater than the internal pressure. From the plot in Fig. 2 showing the variation of the tangential stress on the inner surface of the cylinder against the values of the ratios of E's and τ's, the rapid increase in the magnitude as the increase in the degree of mate-

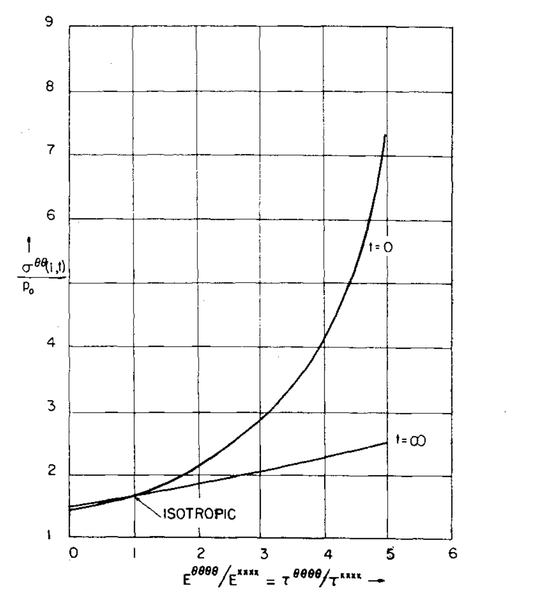

Fig. 2 Variation of tangential stress on the inner surface of the cylinder.

rial anisotropy suggests the possibility as a dominating feature in the cracking of solid propellants during rocket firing.

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Shock Formation in Conical Nozzles

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Evaluation of axially symmetric gas flow in conventional conical nozzles by the method of characteristics has revealed the possibility of shock formation within the nozzle. Negative Mach lines, originating at or just downstream of the junction of the throat profile and the cone, intersect near the axis. Modified computer programs have been run to determine the nature of this phenomenon, and it is concluded that the effect is real. It is shown that the shock formation can be removed by changes in the wall contour near the junction.

Introduction

In a recent paper by Migdal and Landis,[1] the performance of supersonic conical nozzles was investigated by application of the method of characteristics. The authors of this present note began a similar program of work during 1960, but it soon was found that, with the conventional conical nozzle, consisting of a circular-arc contour joined to an expansion cone, intersections of Mach lines of the same family occurred near the axis, resulting in negative flow angles and reduced axial Mach numbers. The first intersection took place along the negative characteristic originating from, or just downstream of, the junction of the two profiles.

Three explanations were put forward for this phenomenon. Firstly, the effect might be a true one, and shock formation could be taking place near the axis; although there are no discontinuities in nozzle slope at the junction, there is a change in the rate of expansion, and this could lead to intersection of Mach lines. In support of this, it was noted that such a situation does not exist with parallel-flow or Rao type nozzles, for which satisfactory results had been obtained using the same basic calculating techniques. Secondly, the accuracy used in the computations might not be sufficient and this could be leading to cumulative errors. Thirdly, the initial conditions in the transonic region might not have been defined sufficiently well.

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Although these last two explanations were considered improbable because of the site of the intersections, it was decided to run several new computer programs to examine these points. The purpose of this paper is to describe the various modifications tried and to report the results, which have led to the conclusion that shock formation occurs. One possible method of removing the shock by a small change in the wall contour is discussed also.

Application of the Method of Characteristics

The basic method of characteristics used to evaluate steady, irrotational, homentropic flows with axial symmetry is described in many text books.[2] The equations of flow yield relations for the variations in flow properties along two families of characteristic (Mach) lines; if conditions at two points not on the same Mach line are known, it is possible to find conditions at a third point by using the relations in finite difference form, together with an iterative procedure. Special relations are involved for points on the axis or for a resultant point on the nozzle wall.

Similar methods were used throughout this work, except that the number of unknowns was reduced by employing the Mach number in place of the velocity. The required flow parameters at any point were thus the Mach number and the angle of the flow to the nozzle axis, together with, of course, the (constant) ratio of specific heats.

The iterative procedure for determining the coordinates and flow properties at the third point is based upon a first approximation, which uses values at the two known points to find coefficients of differentials, followed by further successive approximations using average coefficients. This latter step can be done in two ways, either by using mean coefficients or coefficients of mean values. Either way is suitable for the conventional type of nozzle, but, where large changes in flow direction occur (as in E-D nozzles), the latter process must be employed; consequently, it was applied throughout this work.

For points on the nozzle axis, special iterative relations, which do not appear in the literature, had to be developed. These were obtained by expanding terms involving the ratio of the tangent of the flow angle and the radial distance from the axis in appropriate Taylor series.

In order to begin the numerical integration, conditions have to be prescribed at a sufficient number of points not lying on the same characteristic. This can be accomplished by analyzing the flow in the region of the throat analytically. Sauer[3] has shown how the shape of the sonic line can be determined from small perturbation theory, and his methods can be extended to give curves of constant Mach number in the throat region. Hall, in a more recent publication,[4] derived expressions from which second- and third-order corrections also can be evaluated.

The methods used have been based mainly on Sauer's first-order theory, although Hall's relations have been applied also. However, the curve of constant Mach number joining the points on the nozzle wall at the throat section could not be used, since opposite characteristics from some of the points on this curve intersect upstream of the curve. To avoid this, conditions were derived along straight lines, joining the points on the nozzle wall at the throat section and the intersection of the constant Mach number parabola through these points with the axis of symmetry. The Mach number and flow direction along these lines then could be computed easily.

Except for cases mentioned in the next section, the following calculating procedure was adopted. Initial conditions were determined at twenty-one equally spaced points between the axis and the throat wall, the values being calculated and stored to the full machine capacity. The pattern used was to calculate along negative characteristics, beginning with the point on the nozzle wall at the throat section. However, after initial trials, negative characteristics from the other starting points were not used, since these led to a congested band of Mach lines and the possibility of errors being intro-